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Design of an Exponentially Weighted Moving Average (EWMA) and An Exponentially Weighted Root Mean Square (EWRMS) Control Chart

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Abstract— *Some of the most widely-used form of control charts Walter Shewhart charts are sensitive to detecting relatively large shifts in the process.*

On Shewhart charts every observation is plotted independently of previous observations.The quantity plotted on Cusum charts include all previous observations;

All of which are given equal weight in calculating the plotted value.

Exponentially Weighted Moving Average (EWMA) charts are a kind of compromise between these two extremes.

The plotted quantity is a weighted average of all the observations to date, but the weights decrease very quickly backwards in time, so that the most recent observations are the main determinants of the current plotting point.[3][4]

A cumulative sums (CUSUM) charts plot the of the deviations of each sample value from a target value. It has been used in various industries (especially the chemical industry)

and the form of the CUSUM has been refined over the years to further increase its sensitivity. Two types of charts are primarily used to detect smaller shifts, namely Cumulative Sum (CUSUM) charts and Exponentially Weighted Moving Average (EWMA) charts. E.S.Page2 (1954) originally developed the CUSUM chart. ***[2][3]***

Geometric moving–average control chart is effective alternatives to the Shewhart control chart may be used when small process shifts are of interest primarily used to detect smaller shifts, namely CUSUM and EWMA charts are excellent alternatives to the Shewhart control chart.

EWMA methodology - developed by S.W. Robert in 1959, he chose the weights to decrease geometrically with the age of the observations, he referred to the control chart based on such a weighting system as a geometric moving –average control chart.

Mac Gregor and Harris (1993) recommend that the square root of EWMS, SQR (EWMS), be plotted. Accordingly, they call the corresponding control chart an exponentially weighted root mean square (EWRMS) chart, The EWRMS statistic will react not only to shifts in the process variance but also shifts in the process mean[4] [8]

Keywords— Exponentially Weighted Moving Average (EWMA)-Chart, exponentially weighted root mean square (EWRMS)-Chart.

I. INTRODUCTION

The control charts namely, Shewart chart (Shewhart, 1924), EWMA chart (Roberts, 1959) and CUSUM chart (Page, 1954) are often used for detecting shifts in a sequence of independent normal observations with common variance coming from a particular process. [1][2][6]

Statistical Quality control methods have a large area of study. In its own right is. Central to success in modern industry with its emphasis on reducing costs while at the same time improving quality, Statistical quality control came from Dr. Walter Shewhart in 1924. He recognized that in a manufacturing process, there will always be variation in the resulting products.

Shewhart developed a simple graphical technique - the control chart - for determining if the product variable is within acceptable limits or not.

The control charts namely, Shewart chart (Shewhart,1924),EWMA chart (Roberts,1959) and CUSUM chart (Page, 1954) are often used for detecting shifts in a sequence of independent normal observations with common variance coming from a particular process. [1][3][5**]**

1-Exponentially Weighted Moving Average(EWMA)

The Exponentially Weighted Moving Average (EWMA) control charts and other sequential approaches, like Cumulative Sum (CUSUM) charts, are an alternative to

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Shewhart control charts and are especially effective in detecting small process shiftsEWMA control chart was introduced by Roberts (1959). [4][5].And defined as:

$$Z_i = \lambda xi + (1-\lambda)z_{i-1} \quad \text{….1}$$

Were $0 < \lambda \le 1$ is a constant and the starting value required with the first sample at i=1 is the process target value, so that $Z_0 = u_o$

Sometimes the average of the data is used to start value then it become$Z_0 = \bar{x}$

If the observations X_i are independent random variables with σ^2 then the variance is:

$$\sigma_{xi}^2 = \sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \quad \text{…. 2}$$

Then the EWMA control chart would be constructed by plotting Z_i versus the sample number I. The center line and control limits for the EWMA control chart are as follows[7][11]:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \text{… 3}$$

$$Ceterline = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2i}]} \quad \text{…. 4}$$

From the UCL, and LCL that the term $[1-(1-\lambda)^{2i}]$ Approaches unity as I get larger, than the ULC and LCL it become as:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{….5}$$

$$Ceterline = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}} \quad \text{…. 6}$$

2-Monitoring Variability of Individual:[3][6][11]

MC Gregore and Harris (1993) discuss the use of EWMA-based statistics for monitoring the process slandered deviation, Let $X_i$be a normally distribution with mean and standard deviation then the exponentially weighted mean square error (EWMS) is:

$S_i^2 = \lambda(X_i - \mu)^2 + (1-\lambda)S_{i-1}^2$ Shown that $E(S_i^2) = \sigma^2$ Has been an approximate to Chi-Square distribution with degrees of freedom $v = (2-\lambda)/\lambda$ the exponentially weighted root mean square (EWRMS) control limits given by:

$$UCL = \sigma_0\sqrt{\frac{\chi^2_{v.\alpha/2}}{v}} \quad \text{…. 7}$$

$$LCL = \sigma_0\sqrt{\frac{\chi^2_{v,1-(\frac{\alpha}{2})}}{v}} \quad \text{…. 8}$$

And the Exponentially weighted moving variance ($EWMV_{zi}$) as:

$$S_i^2 = \lambda(X_i - Zi)^2 + (1-\lambda)S_{i-1}^2 \quad \text{…. 9}$$

It is possible to construct Cusum control charts for monitoring process variability.

Consider a process with its quality characteristic Xi distributed as normal with mean μ and variance σ2 . Then the standardized random variable The standardized random variable is :

$Z_i = (X_i - \mu_0)/\sigma$,

Hawkins (1981-1993) showed that the random variable$\sqrt{|Z_i|}$Is approximately distributed as normal with mean 0.822 and standard deviation 0.349; that is, the random variable As show us:

$$V_i = \frac{\sqrt{|Z_i|}-0.822}{0.349} \quad \text{…. 10}$$

Is distributed as standard normal. Furthermore, when the variance of Xi increases, the mean of Vi will increase. However, any change in the mean of Vi can be detected by designing a two- sided CUSUM - Chart, Where Vi is approximately N(0,1).The exponentially weighted moving variance(EWMV) and the exponentially weighted root mean square(EWRMS) charts to monitor the variation of the process that generates an individual autocorrelated observation.The (EWRMS) statistic uses the squared deviation of observation from the known process mean or from the target value. The (EWMV) statistic uses the squared deviations of the observations from an estimate of the process mean. [3][8][9][10]

The degree of freedom is related to exponential weighing constant λ as follows;

$$f = \frac{2-\lambda}{\lambda} \quad \text{….. 11}$$

II. NUMERICAL ILLUSTRATION:

In this paper highlight the positive measurement features of above proposed EWMA and EWRMS control limit. Then drawing charts of both control limits to compare the numerical result analysis and charts. For that In this case use the data of chemical analysis on (Loss on ignition L.O.I) from cement. The data are forty sample with a sample size of (4) the total measurement number is (160). As shows in table (1).

Table.1: Data of L.O.I and EWMA

#	X1	X2	X3	X4	Mean	Rang	EWMA
1	3.66	3.11	3.66	3.04	3.3675	0.62	2.5
2	3.65	3.44	3.65	2.63	3.3425	1.02	2.67
3	3.71	3.53	3.71	2.6	3.3875	1.11	2.81
4	3.54	3.69	3.62	2.54	3.3475	1.15	2.92
5	3.63	3.87	3.55	2.67	3.43	1.2	3.01
6	3.69	3.48	3.14	2.86	3.2925	0.83	3.09
7	3.62	3.47	3.58	2.78	3.3625	0.84	3.13

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8	3.55	3.83	3.53	2.8	3.4275	1.03	3.18
9	3.14	3.84	3.57	2.87	3.355	0.97	3.23
10	3.58	3.81	3.62	2.61	3.405	1.2	3.25
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
35	3.01	2.05	2.13	2.63	2.455	0.96	2.59
36	2.97	1.83	2.37	2.58	2.4375	1.14	2.56
37	2.71	2.26	2.68	2.61	2.565	0.45	2.56
38	2.91	2.11	2.74	2.96	2.68	0.85	2.58
39	3.12	2.1	2.63	2.85	2.675	1.02	2.60
40	3.13	2.53	2.8	2.46	2.73	0.67	2.63

Determining the EWMA value of data in a table (1) by using Eq. (5 and 6). Thevalue of upper limits is (3.464), the lower limit is (2.54), and central limit is (3), where the target value is (2.5). Fig (1) shows the EWMA control chart. From the control chart it shows that the first (17) point between central and upper limits.The c value is (3.605), and the first value is (3.3675), after that point the value of EWMA falling down to the lower limits asshown in the fig(1)

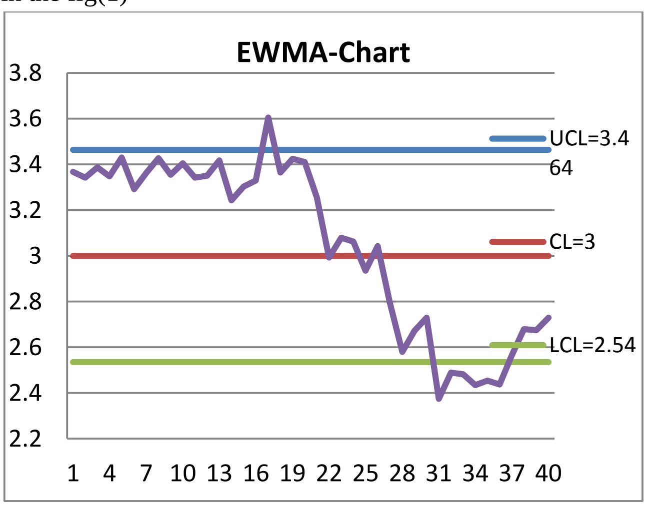

Fig.1: EWMA-Chart

Table.2: Value of EWMAvi and EWRMS

	Vi	EWMA Vi	EWMA s	EWRMS
1	1.419	0.284	0.327	0.571
2	1.364	0.500	0.403	0.635
3	1.462	0.692	0.480	0.693
4	1.375	0.829	0.528	0.726
5	1.552	0.974	0.595	0.771
6	1.252	1.029	0.602	0.776
7	1.408	1.105	0.630	0.794
8	1.547	1.193	0.676	0.822
9	1.392	1.233	0.687	0.829
10	1.500	1.286	0.714	0.845
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
35	-1.496	-1.024	0.069	0.262
36	-1.342	-1.088	0.056	0.236
37	-1.322	-1.135	0.045	0.213
38	-0.636	-1.035	0.043	0.207
39	-0.660	-0.960	0.040	0.201
40	-0.412	-0.850	0.043	0.207

Also By using the data incoming in table (1) and Eq (7 ,8 and 10) it is a Possible to calculate the value of VI, EWMAs and EWRMS as shows in table (2). Then determine upper and lower control limit value of the EWRMS as shows in fig (2), Upper control limits are (0.73) and Lower Limit is (0.2). This chart quite definitively indicates the output of control from the point (5) up to (25), Chart changed direction towards lower of the points 25, this exponentially chart is weighted by the mean square error.

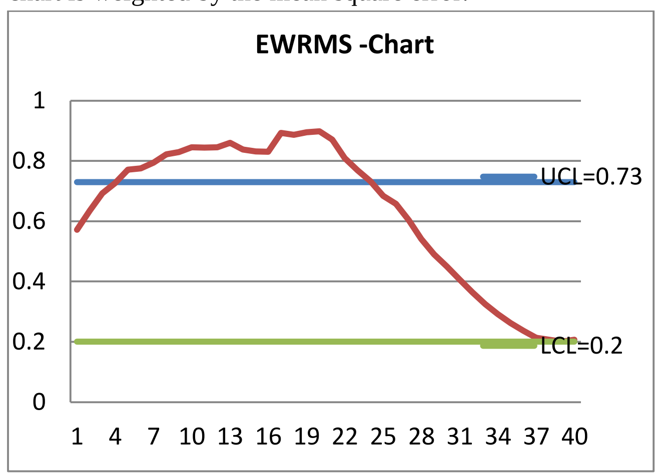

Fig.2: EWRMS-Chart

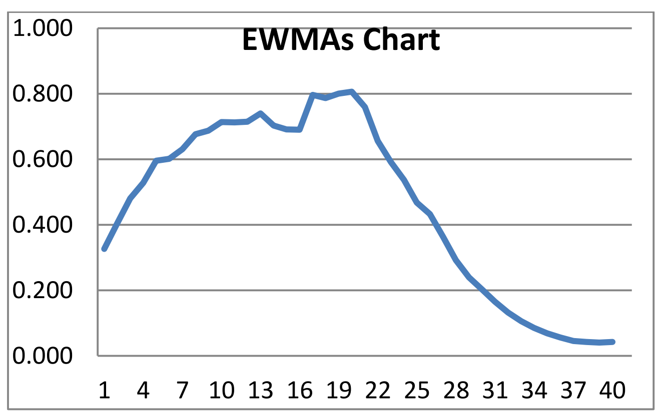

Fig.3: EWMAs-Chart

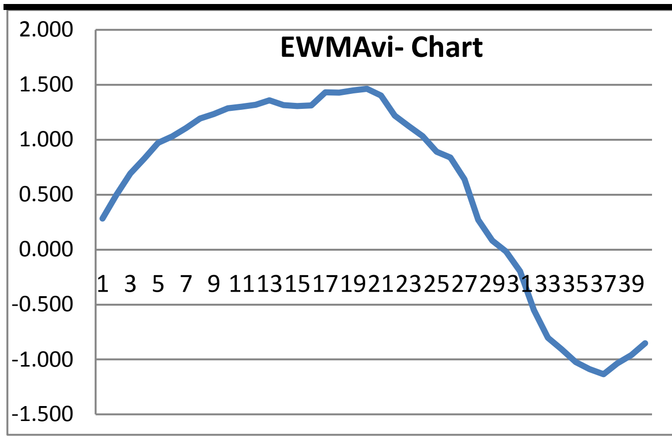

Fig.4: EWMAvi-Chart

Fig (3) is EWMAs-Chart and Fig (4) is EWMAv - Chart, from the both chart shows that the first (20)points located towards upper within the of control limits, after the point(20) change the direction of the chart towards to the lower, the maximum Value of EWMAv is (1.462) and the minimum value is (- 0.850) less than zero. During the comparison between two Cusum charts it shows that the fig.(4) more clear than fig. (3).

III. SUMMARY AND CONCLUSIONS

In this paper achieves limitations of the traditional concept of Cusum, EWMA and EWRMS clear that the planned EWRMS is a good technique between the note in contrast transition process in the process means.

Leaning in a similar way to respond both mean and variance changes, The EWRMS statistic is a good sensitive to shifts the process variance and also shifts in the process mean.

And EWMV-Chart which allows for changing mean it is clear in the event of a control signal, if the signal each of the graphs, we suspect that the change in the average minimum happened with the possibility of both mean and variance having changed.

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